

eg. 1. A 105kg student runs up 23 stairs 12.0 cm high and 17.0 cm wide in 3.0 s.

a) how much work is done by the student?

work $W = Fd \cos \theta$

$d = \text{up}$ because you are pushing against gravity

$d_{\text{up}} = 0.120 \text{ m} \times 23 \text{ stairs} = 0.12 \times 23 = 2.76 \text{ m}$

$W = (105 \text{ kg} \times 9.80 \text{ N/kg}) \times 2.76 \text{ m}$

$105 \times 9.8 \times 2.76 = 2,840.04 = 2.8 \text{ kJ}$

b) how much work is done by gravity?

since F is opposite d , the work is negative

$W = -2.8 \text{ kJ}$

c) how much work is done by the normal force

work = 0 because the normal force has no displacement

d) What is the students power, $P = W/t$

Watt = 1 J/s or horsepower, 1 hp = 746W

Work is measured in Joules, $J = \text{Nm}$

is Work a scalar or vector? a scalar - no direction

$P = W/t = 2840 \text{ J} / 3.0 \text{ s}$

$2840 / 3 = 946.6667 = 9.5 \times 10^2 \text{ W}$

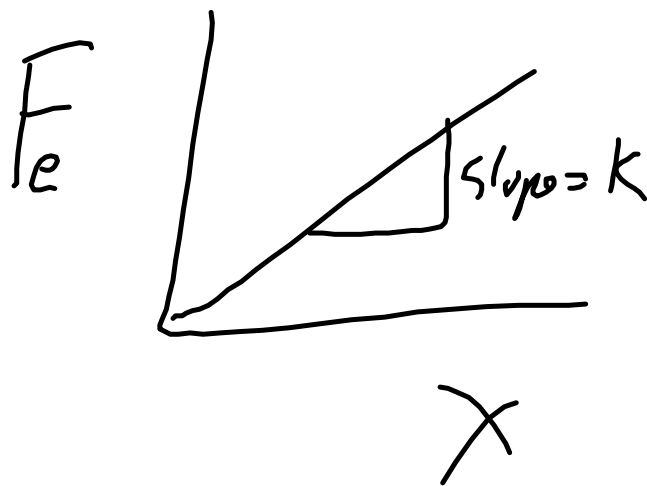
$946.6667 / 746 = 1.269 = 1.3 \text{ horsepower, hp}$

2. If you pull a spring sideways with 6.0 N of force it stretches 3.0 cm. If you pull it until 12.0N of

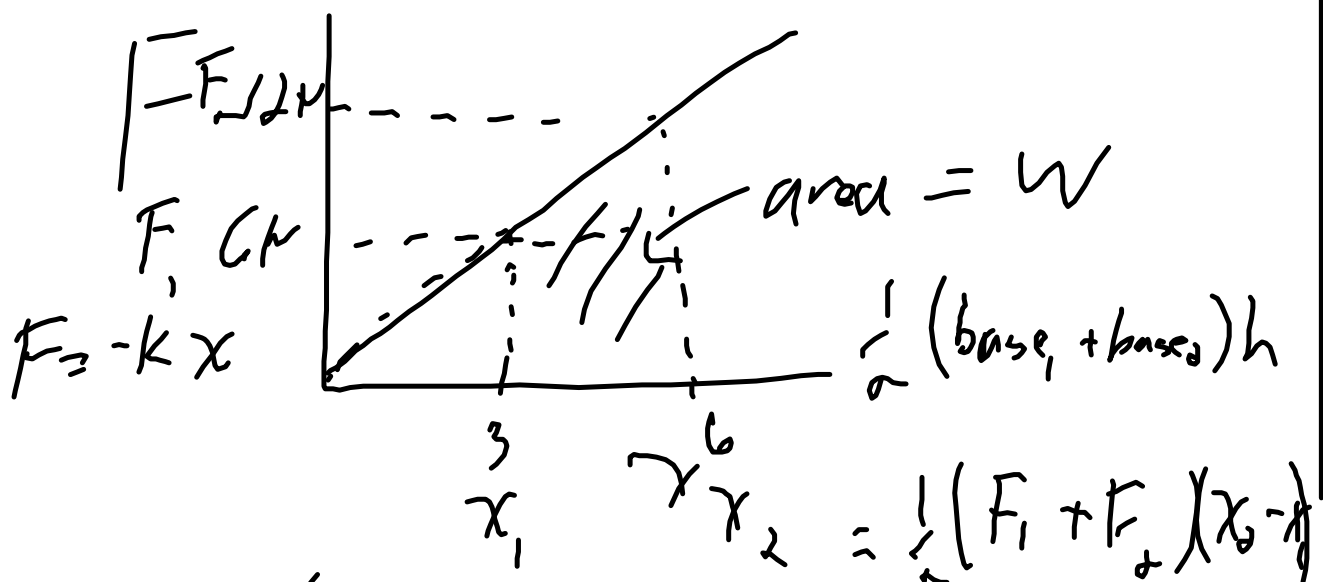
force stretches 6.0 cm. How much work is done on the spring pulling it from 3.0 cm to 6.0 cm? (area under the F-d graph = W)

We can't use $W = Fd \cos \theta$ because F is not constant - applied force can be any value, air drag changes with velocity - gravity far from Earth changes - so we use calculus to derive the equation

Hooke's Law



F_e elastic force, Spring
k is elastic constant, in $\frac{N}{m}$



$$W = \frac{1}{2} (kx_1 + kx_2) (x_2 - x_1)$$

$$W = \underline{\frac{1}{2} k x_2^2} - \underline{\frac{1}{2} k x_1^2}$$

$$W = \Delta \text{Energy}$$

$$\text{elastic energy} = \boxed{E_{\text{elastic}} = \frac{1}{2} k x^2}$$

$$k = \frac{F}{x} = \frac{6 \text{ N}}{3 \text{ cm}} = 2.0 \frac{\text{N}}{\text{cm}}$$

$$k = 2.0 \times 10^2 \frac{\text{N}}{\text{m}}$$

$$\begin{aligned} W &= \frac{1}{2} \left(2 \times 10^2 \frac{\text{N}}{\text{m}} \right) (0.0 \text{ m})^2 - \\ &\quad \frac{1}{2} \left(2 \times 10^2 \frac{\text{N}}{\text{m}} \right) (0.03 \text{ m})^2 \\ &= \boxed{0.27 \text{ J}} \end{aligned}$$

So this is how we can derive equations for different types of energy.

use $W = Fd \cos \theta$ = area under the F-d graph
and $W = \text{change in energy}$

Energy stored in a spring or elastic object
 $E_{\text{elastic}} = 1/2 kx^2$ where k =elastic constant $|F/x|$

How about kinetic energy? E_k is the energy of motion. Changes in motion are caused by the net force on the object.

$$W_{\text{net}} = \Delta E_k = F_{\text{net}} d = mad \quad 2ad = v_f^2 - v_i^2$$

$$ad = 1/2(v_f^2 - v_i^2)$$

$$\Delta E_k = m \cdot 1/2(v_f^2 - v_i^2) = 1/2mv_f^2 - 1/2mv_i^2$$

$$E_k = 1/2mv^2$$

units: Joules, J

scalar - watch out! don't get it confused with momentum - a vector

Gravitational energy, E_g

$$W_g = \Delta E_g$$

work done against gravity = change in gravitational energy

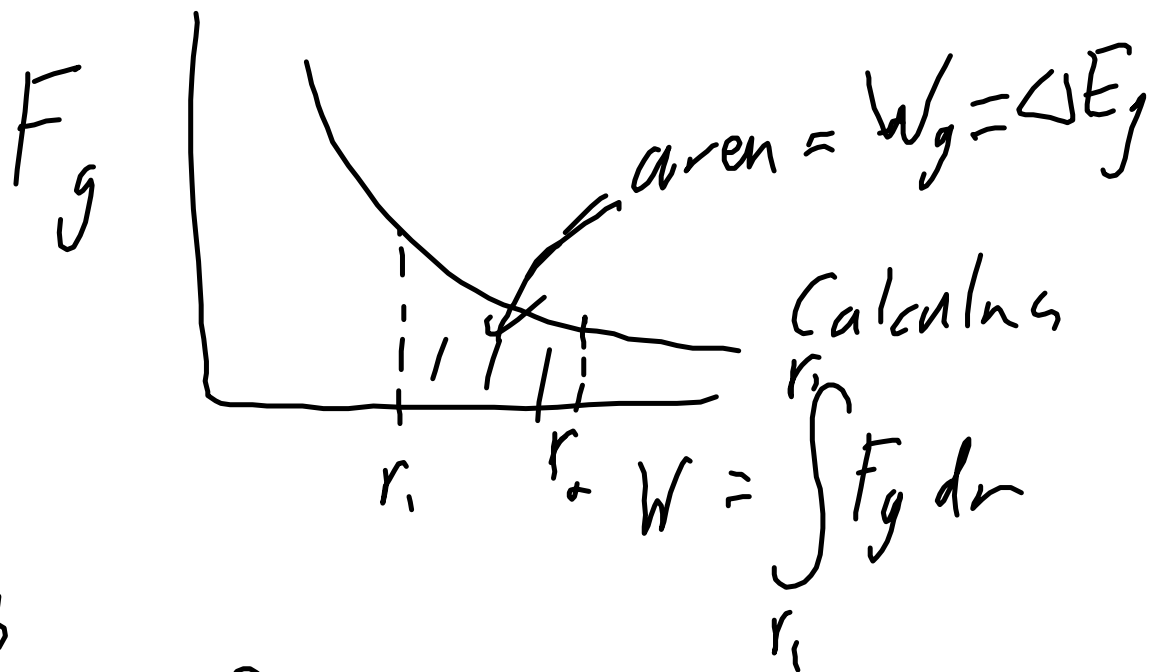
uniform gravitational field, g

$$W_g = \Delta E_g = F_g d = mg\Delta h$$

$E_g = mgh$ relative to any arbitrary reference point $h=0$.

keeners - try for next class

$$F_g = GMm/r^2$$



$$\frac{d x^3}{d x} = 3 x^2$$

$$\int x^2 = \frac{1}{3} x^3$$

Big Idea

Energy is conserved

add up the total energy before

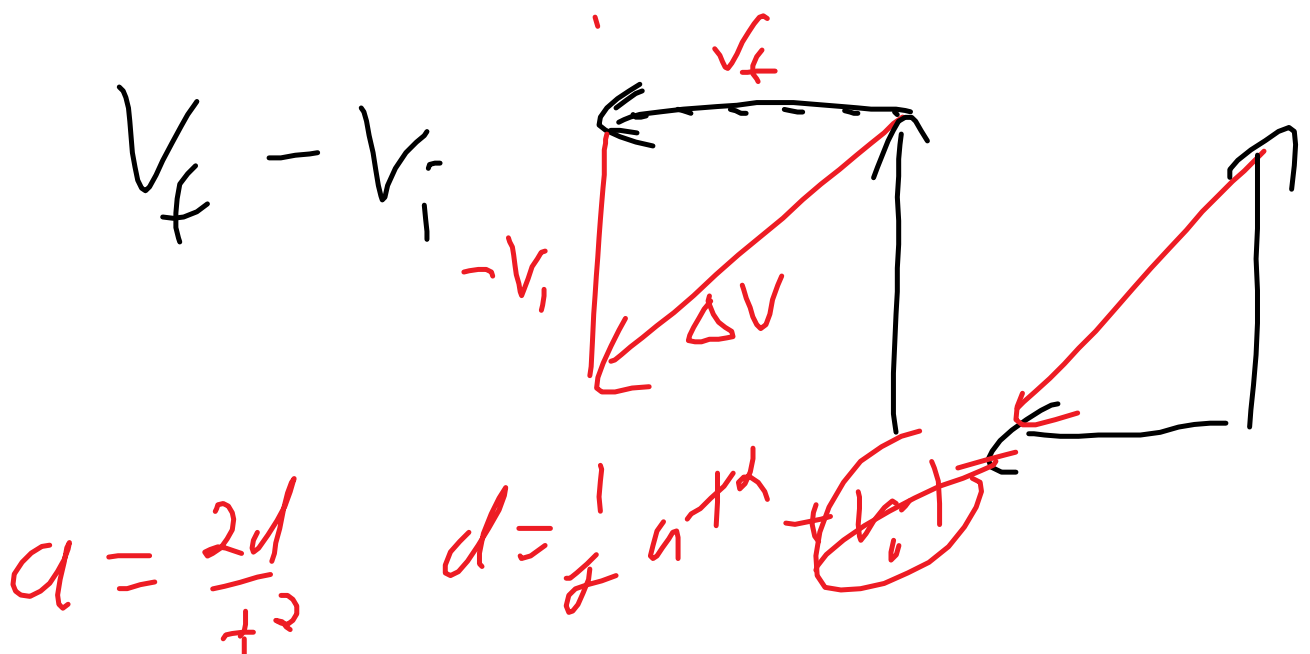
= total energy after

independent of path or other issues.

eg. A 700.0 kg car is moving at 5.0 m/s at the top of a hill, height of 10.0 m. The road is so icy, that it is essentially frictionless.

a) what is the kinetic energy of the car at the top of the hill?

- b) what is the gravitational energy of the car at the top?
- c) what is the total energy of the car (some books call it the mechanical energy)
- d) if no energy is lost as the car slides down, what is the speed of the car at the bottom of the hill?
- e) the car slides up another hill 5.0m high. What is the speed at the top of that hill?
- f) if the car is moving at 4.0 m/s at the top of the second hill, how much energy was lost in frictional force?
- g) write a paragraph describing how to drive in the snow given you knowledge of physics.
 - coefficient of static friction / kinetic
 - $F=ma$
 - inertia
 - kinetic and gravitational energy
 - melting point of ice/pressure/friction?



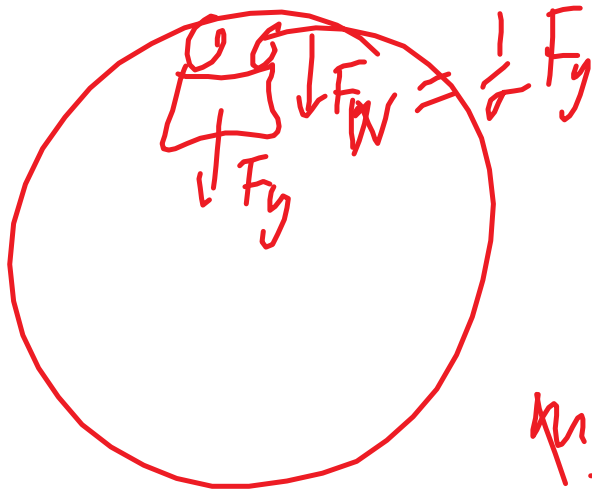
$$a = \frac{2d}{t^2}$$

$$d = \frac{1}{2} a t^2 + v_0 t$$



$$F_T - F_g = ma$$

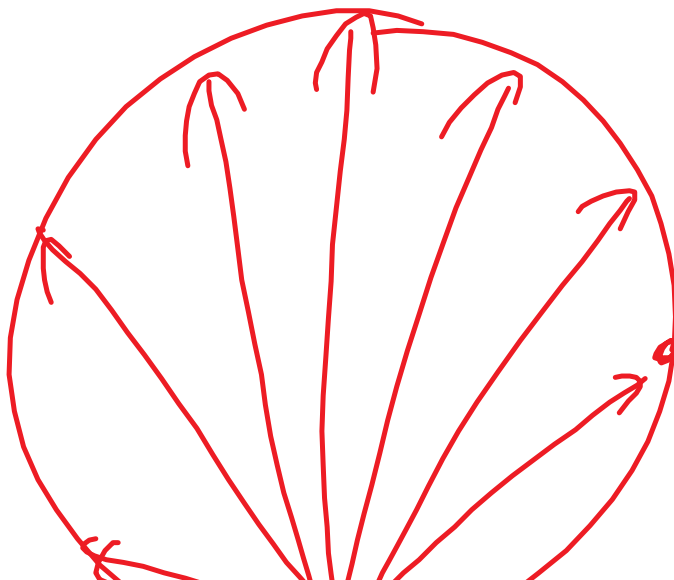
$$F_T = mg + m \frac{2d}{t^2}$$



$$F_c = F_g + F_W$$

$$m \frac{v^2}{r} = mg + \frac{1}{2} mg$$

$$v = \sqrt{1.5gr}$$





$$2 \uparrow F_T = 28 + 28 = \underline{56 \text{ N}}$$



$$F_N = F_g - F_c$$

$$\frac{1}{4} F_g$$



$$F_N = F_g + F_c$$

Block 2-4

questions about work

derive energy equations

term 1 test

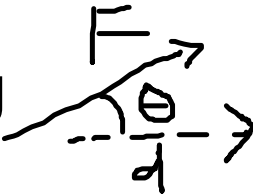
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a) how much work is done by the student.

$$W = Fd \cos \theta$$

units: Joules, J

scalar



$$d_{\text{up}} = 23 \times 0.12 = 2.76 \text{ m}$$

$$F = mg$$

$$W = mgd = 105 \times 9.8 \times 2.76 = 2,840.04$$

$$W = 2.8 \text{ kJ}$$

ignore the sideways motion

a) how much work is done by gravity

F is opposite d , but the magnitudes are the same as previous question, so

$$W = -2.8 \text{ kJ}$$

a) how much work is done by the normal force

the normal force does not act through the displacement, $W = 0$

when does the normal force do work? in an elevator, back of a car seat,

b) What is the students power, $P=W/t$

Watt = 1 J/s or horsepower, 1 hp = 746W

Work is measured in Joules, $J=Nm$

is Work a scalar or vector? a scalar - no direction

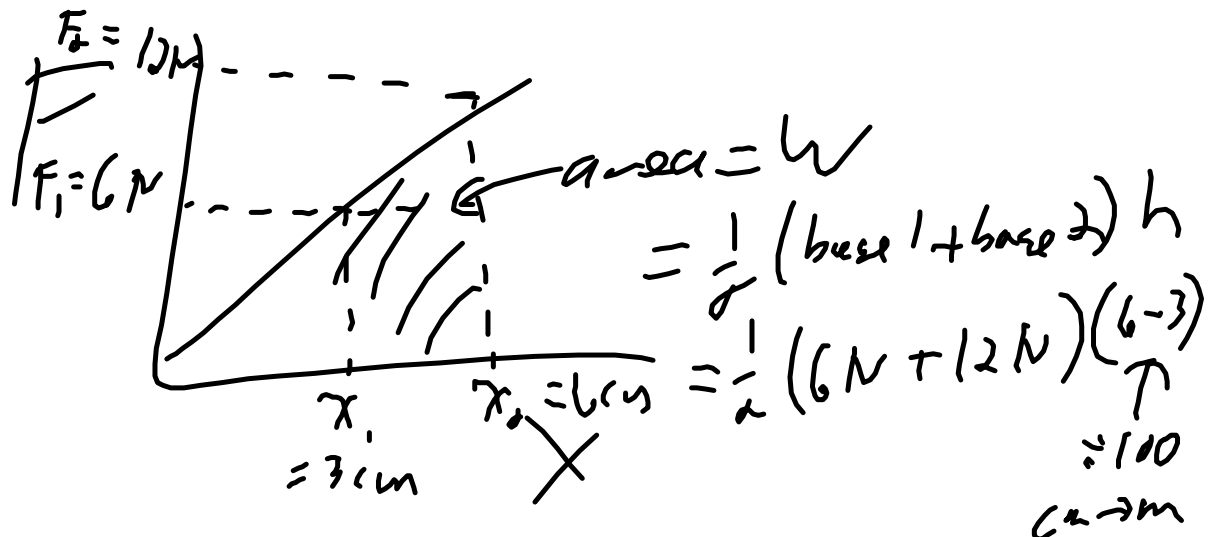
$$P=W/t = 2840/3=946.6667 \text{ W}$$

$$946.66667/746=1.26899 \text{ hp}$$

$$P=9.5 \times 10^2 \text{ W or } 1.3 \text{ horsepower}$$

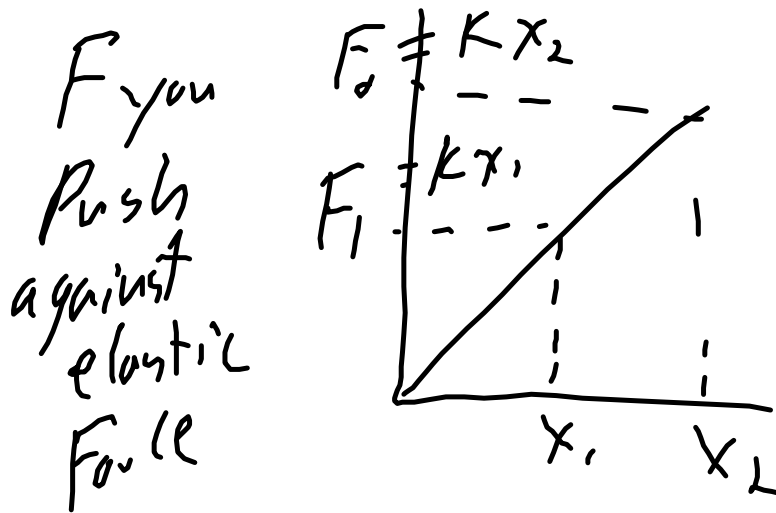
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$W=Fdcos\theta$ but if F is not constant, we use the area under the F-d graph (= integral in calculus)



$$W = 0.27 \text{ J}$$

Hooke's Law $F_{\text{elastic}} = -kx$



the change in elastic energy = Work done against the spring = area under the graph

$$\frac{1}{2}(F_1 + F_2)(x_2 - x_1) = \frac{1}{2}(kx_1 + kx_2)(x_2 - x_1)$$

k is the elastic constant, in N/m

$$\frac{1}{2} k(x_1 + x_2)(x_2 - x_1)$$

$\frac{1}{2} k(x_2^2 - x_1^2) = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$ = the change in elastic energy, so the elastic energy at any point, $E_{\text{elastic}} = \frac{1}{2} kx^2$

so we can resolve the previous question:

$$E = \frac{1}{2} kx^2 \quad k = |F/x| = 6 \text{ N} / 3 \text{ cm} = 2.0 \text{ N/cm} = 200 \text{ N/m}$$

$$E = \frac{1}{2} 200 \text{ N/m} \times (0.03 \text{ m})^2 =$$

$$0.03 \times 0.03 \times 100 = 0.09 \text{ J}$$

$$E = \frac{1}{2} 200 \text{ N/m} \times (0.06 \text{ m})^2$$

$$0.06 \times 0.06 \times 100 = 0.36$$

$0.36 - 0.09 = 0.27 \text{ J}$ same answer but using change in elastic energy

calculus: $\frac{d}{dx} x^3 = 3x^2$ $\frac{dx^3}{dx} = 3x^2$

$$\int x^2 dx = \frac{1}{3} x^3$$

$$\int F_{\text{elastic}} dx$$

$$\int kx dx$$

$$\frac{1}{2} kx^2$$

Keeney

$$\int F_g dr \quad F_g = \frac{GMm}{r^2}$$

I will give you the equations, so you don't need calculus but it's fun.

Derive equations for other types of energy.
Work done against gravity = change in

gravitational energy

$$W_g = \Delta E_g$$

$$= F_g d = mg \Delta h = \Delta E_g$$

so $E_g = mgh$ where h is relative to any arbitrary reference point, $h=0$ - the lowest point.

Kinetic energy, E_k energy of motion

Work done by the net force = change in kinetic energy.

$$W_{\text{net}} = \Delta E_k$$

$$= F_{\text{net}} d = mad \quad 2ad = v_f^2 - v_i^2 \text{ (kinematics)}$$

$$W_{\text{net}} = m \frac{1}{2}(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$E_k = \frac{1}{2}mv^2$$